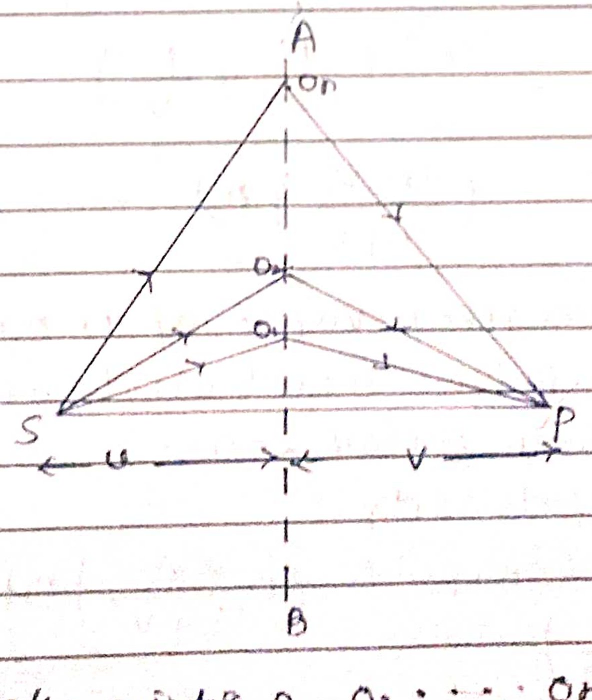
a) Zone plate : A zone plate is a specially constructed screen such that light is abstructed from every alternate zone. It consists of a glass plate having concentric circles of radoi proportional to the square root of natural numbers 1, 2, 3, ... Now even or odd annular spaces between the two circles are blackened to make them apaque.

We consider AB as a zone plate placed perpendicular to the plane of fig. S be a monochromatic light Source of wavelength λ having prependicular distance SO = u from the zone plate. Let we have to determine resultant intensity at P, where OP = v.



Now we take points O_1, O_2 ... O_n on the plate such that

$$SO_1 + O_1P = SO + OP + \frac{\lambda}{2} = u + v + \frac{\lambda}{2}$$

$$SO_2 + O_2P = SO + OP + 2 \cdot \frac{\lambda}{2} = u + v + \frac{2\lambda}{2}$$

. . . .

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And, $SO_n + O_nP = SO + OP + n \cdot \frac{\lambda}{2} = u + v + n \cdot \frac{\lambda}{2}$

Now, we draw concentric circles of radii

$OO_1 = r_1,\ OO_2 = r_2, \ldots, OO_n = r_n$ on the plate

Now, $SO_n^2 = SO^2 + OO_n^2$

$$\therefore SO_n = (u^2 + r_n^2)^{1/2} = u\left(1 + \frac{r_n^2}{u^2}\right)^{\frac{1}{2}}$$

$$= u\left(1 + \frac{1}{2}\frac{r_n^2}{u^2}\right) \quad \left[\text{As, } \frac{r_n^2}{u^2} << 1\right]$$

Similarly, $PO_n = v\left[1 + \frac{1}{2}\frac{r_n^2}{v^2}\right]$

$$\therefore SO_n + O_nP = u + v + \frac{n\lambda}{2} = u\left(1 + \frac{1}{2}\frac{r_n^2}{u^2}\right) + v\left(1 + \frac{1}{2}\frac{r_n^2}{v^2}\right)$$

$$= u + v + \frac{1}{2} r_n^2 \left(\frac{1}{u} + \frac{1}{v}\right)$$

or, $\frac{n\lambda}{2} = \frac{r_n^2}{2}\left(\frac{1}{u} + \frac{1}{v}\right)$

$$\therefore r_n^2 = \left(\frac{u \cdot v}{u + v}\right) n\lambda \quad \text{——} \ (1)$$

Thus for a given value of u and v, $r_n \propto \sqrt{n}$

If, A_n = Area between last circle and last but one circle = nth half period zone.

Resultant intensity :

$$A_n = \pi r_n^2 - \pi (r_{n-1})^2 = \left\{\left(\frac{uv}{u+v}\right) n\lambda - \left(\frac{uv}{u+v}\right)(n-1)\lambda\right\}$$

$$= n\lambda\left(\frac{uv}{u+v}\right)$$

Thus A_n is independent of n. Hence areas of all annular zones are equal. Thus numerical values of displacements $R_1, R_2 \cdots$ due to 1st, 2nd ... zone diminish slowly with increase of n. As the displacement from alternate zones will have opposite phases, the resultant displacement R at P is given by -

$$R = R_1 - R_2 + R_3 - \cdots$$

$$= \frac{R_1}{2} + \left(\frac{R_1}{2} - R_2 + \frac{R_3}{2}\right) + \left(\frac{R_3}{2} - R_4 - \frac{R_5}{2}\right) + \cdots$$

$$= \frac{R_1}{2}, \text{ when } n \text{ is very large.}$$

If we stop the secondary waves from second, fourth, sixth, ... zones then, $R = R_1 + R_3 + R_5 + \cdots$ which is many times greater than the resultant displacement due to all zones.

So to construct a zone plate it is only necessary to draw on a white paper, a large number of concentric circles with radii proportional to square roots of natural numbers and then to blacken the alternate zones. A reduced photograph on a glass plate of this drawing constitute a zone plate.